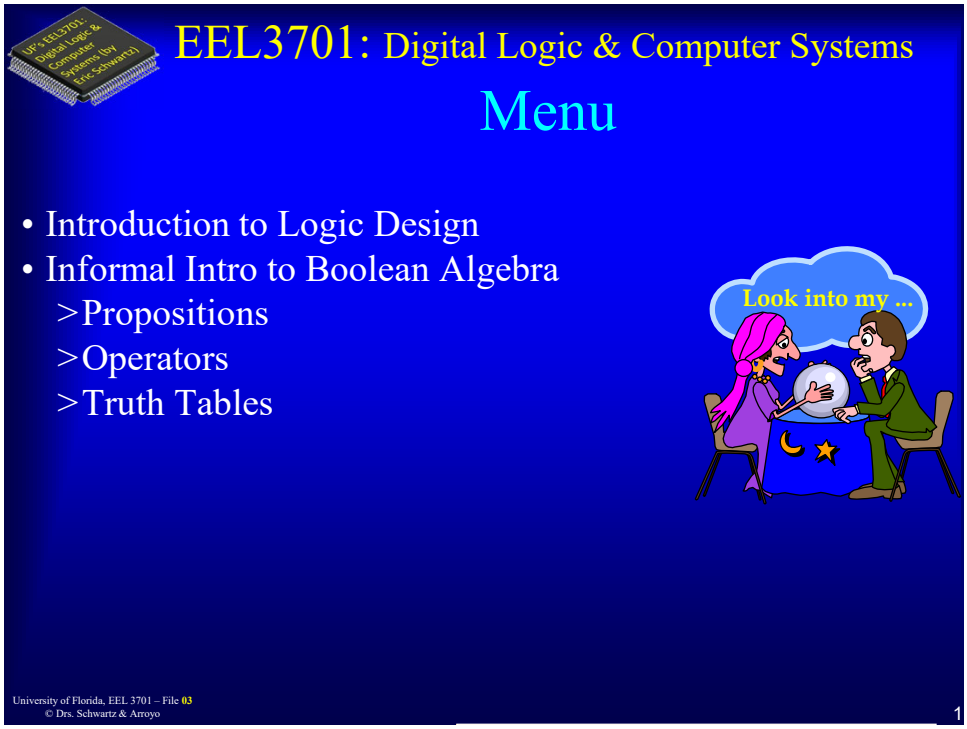
EEL3701: Digital Logic & Computer Systems
Menu
• Introduction to Logic Design
• Informal Intro to Boolean Algebra
>Propositions
>Operators
>Truth Tables
Look into my ...
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1


EEL3701: Digital Logic & Computer Systems
Introduction to Logic Design
• Let's logically analyze a simple paragraph:
When I forget my umbrella and it rains, I get wet. I also get wet when I stick my foot in the bathtub when it has already been filled.
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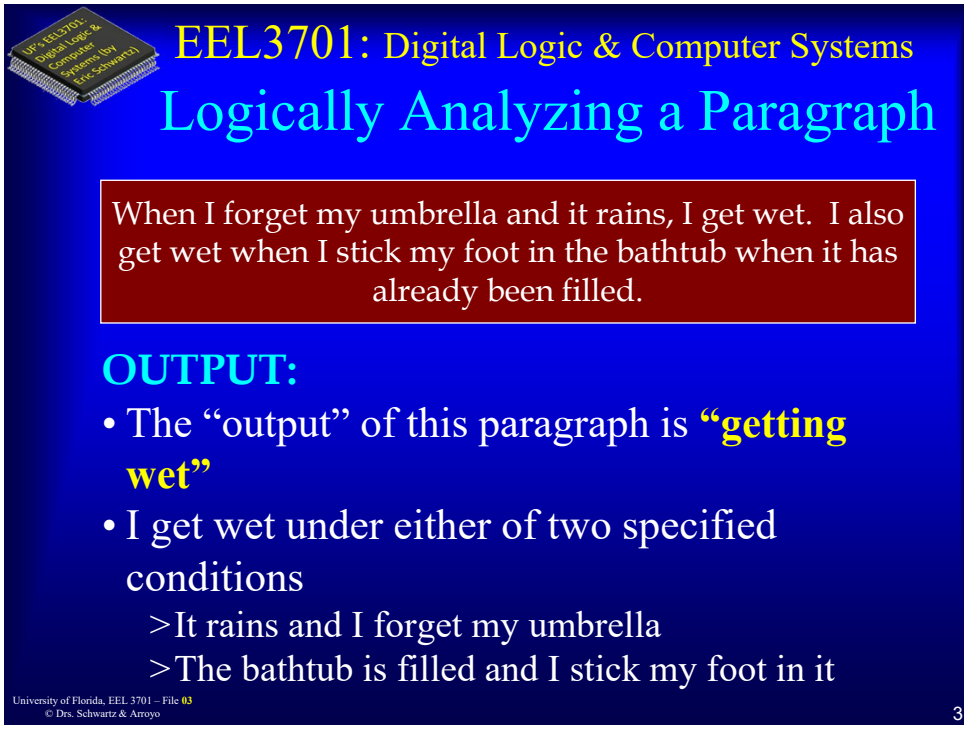
EEL3701: Digital Logic & Computer Systems
Logically Analyzing a Paragraph
When I forget my umbrella and it rains, I get wet. I also get wet when I stick my foot in the bathtub when it has already been filled.
OUTPUT:
• The "output" of this paragraph is "getting wet"
• I get wet under either of two specified conditions
>It rains and I forget my umbrella
>The bathtub is filled and I stick my foot in it
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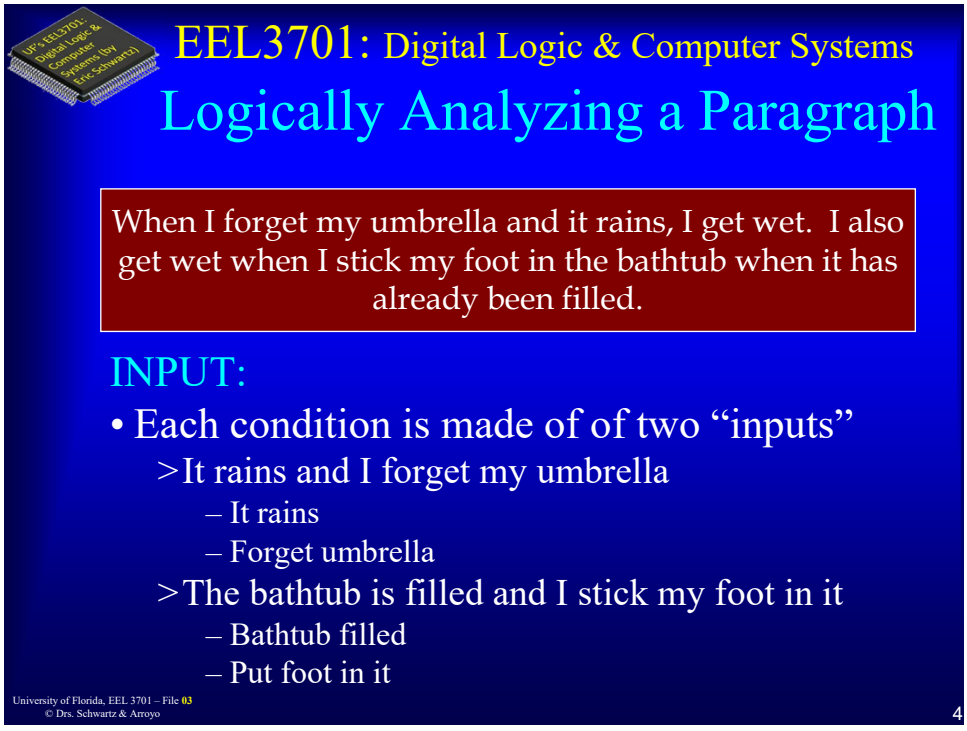
EEL3701: Digital Logic & Computer Systems
Logically Analyzing a Paragraph
When I forget my umbrella and it rains, I get wet. I also get wet when I stick my foot in the bathtub when it has already been filled.
INPUT:
• Each condition is made of of two "inputs"
>It rains and I forget my umbrella
– It rains
– Forget umbrella
>The bathtub is filled and I stick my foot in it
– Bathtub filled
– Put foot in it
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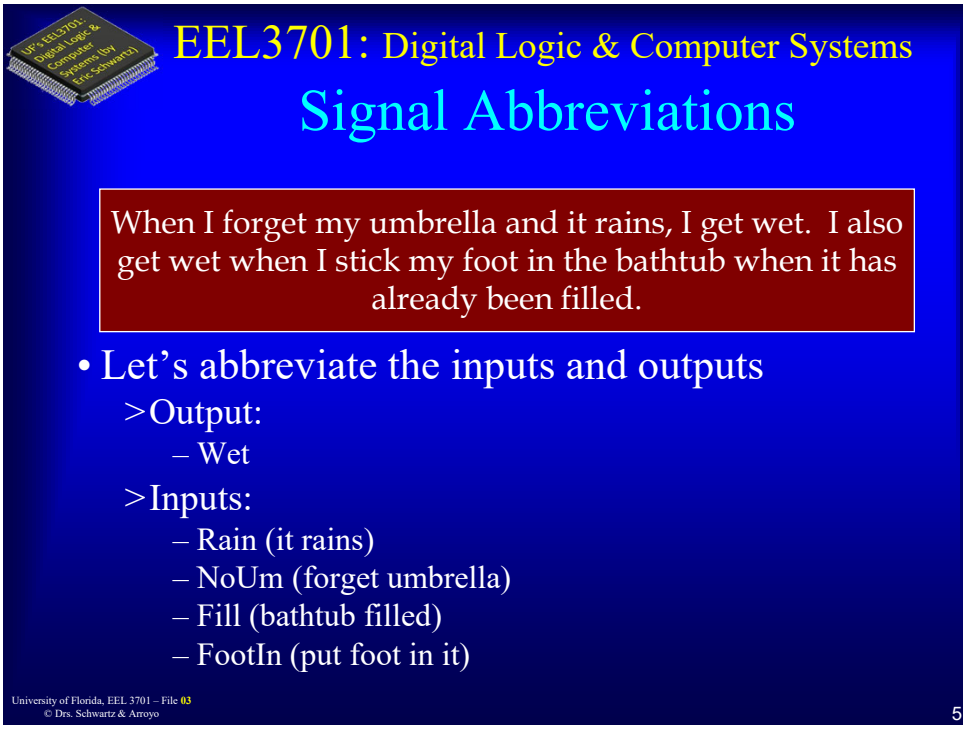
EEL3701: Digital Logic & Computer Systems
Signal Abbreviations
When I forget my umbrella and it rains, I get wet. I also get wet when I stick my foot in the bathtub when it has already been filled.
• Let's abbreviate the inputs and outputs
>Output:
– Wet
>Inputs:
– Rain (it rains)
– NoUm (forget umbrella)
– Fill (bathtub filled)
– FootIn (put foot in it)
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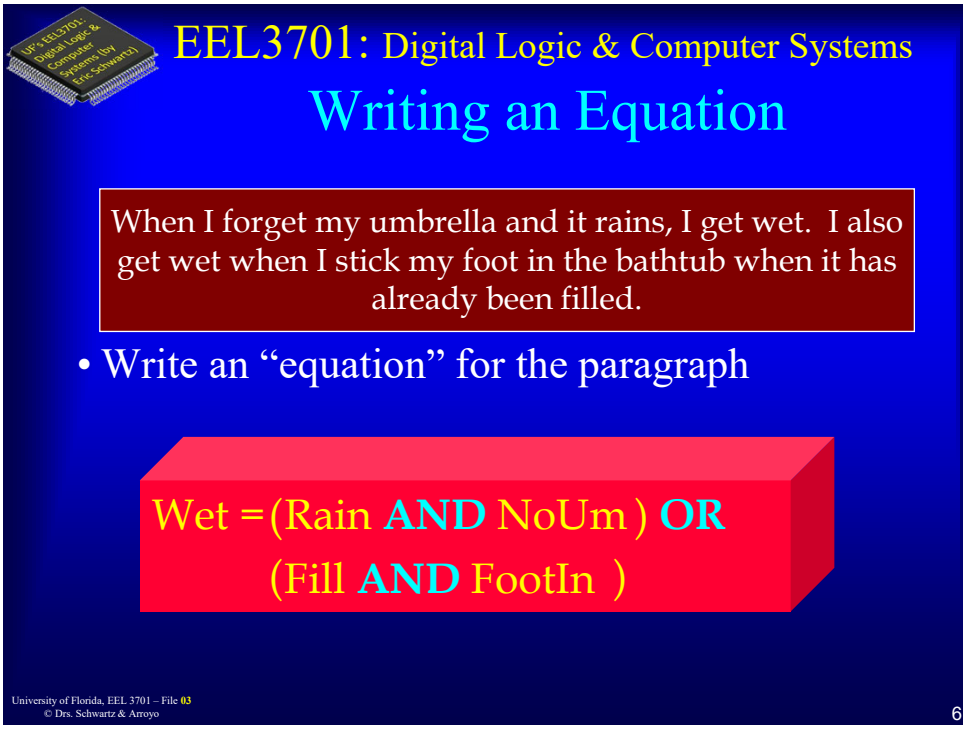
EEL3701: Digital Logic & Computer Systems
Writing an Equation
When I forget my umbrella and it rains, I get wet. I also get wet when I stick my foot in the bathtub when it has already been filled.
• Write an "equation" for the paragraph
Wet = (Rain AND NoUm) OR
(Fill AND FootIn)
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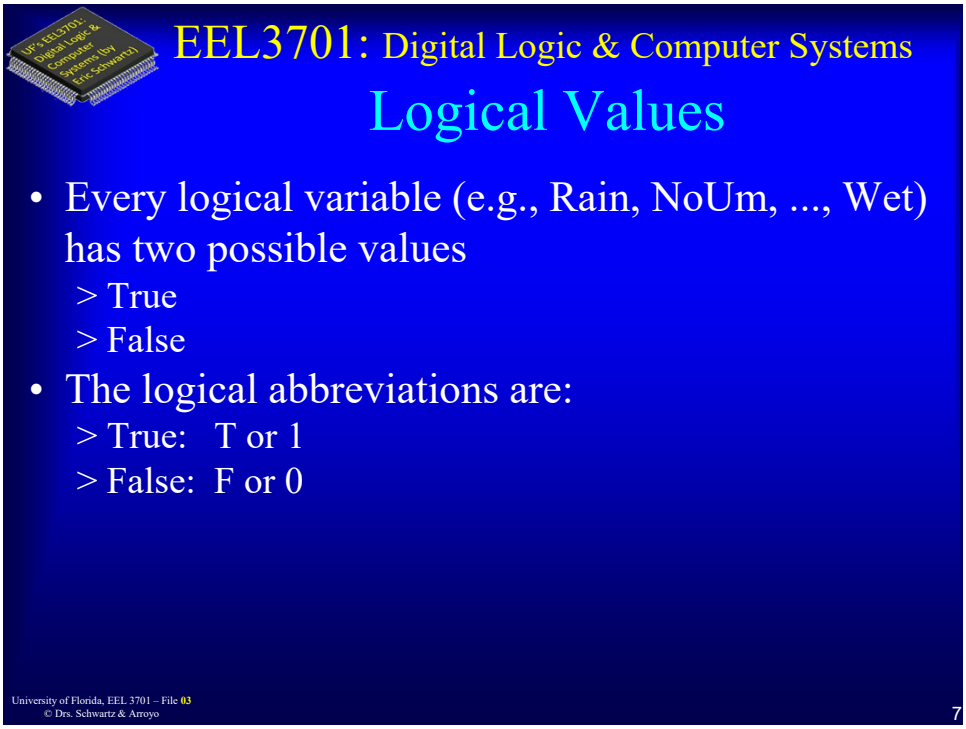
EEL3701: Digital Logic & Computer Systems
Logical Values
• Every logical variable (e.g., Rain, NoUm, ..., Wet) has two possible values
> True
> False
• The logical abbreviations are:
> True: T or 1
> False: F or 0
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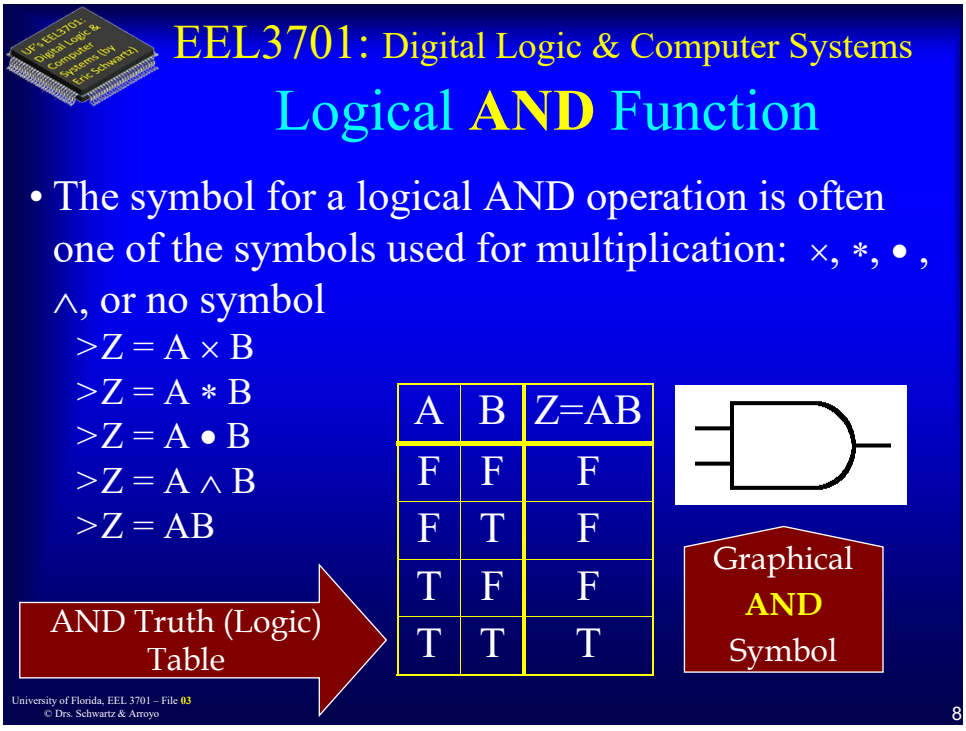
EEL3701: Digital Logic & Computer Systems
Logical AND Function
• The symbol for a logical AND operation is often one of the symbols used for multiplication: ×, *, • , ∧, or no symbol
>Z = A × B
>Z = A * B
>Z = A • B
>Z = A ∧ B
>Z = AB
AND Truth (Logic) Table
A B Z=AB
F F F
F T F
T F F
T T T
Graphical AND Symbol
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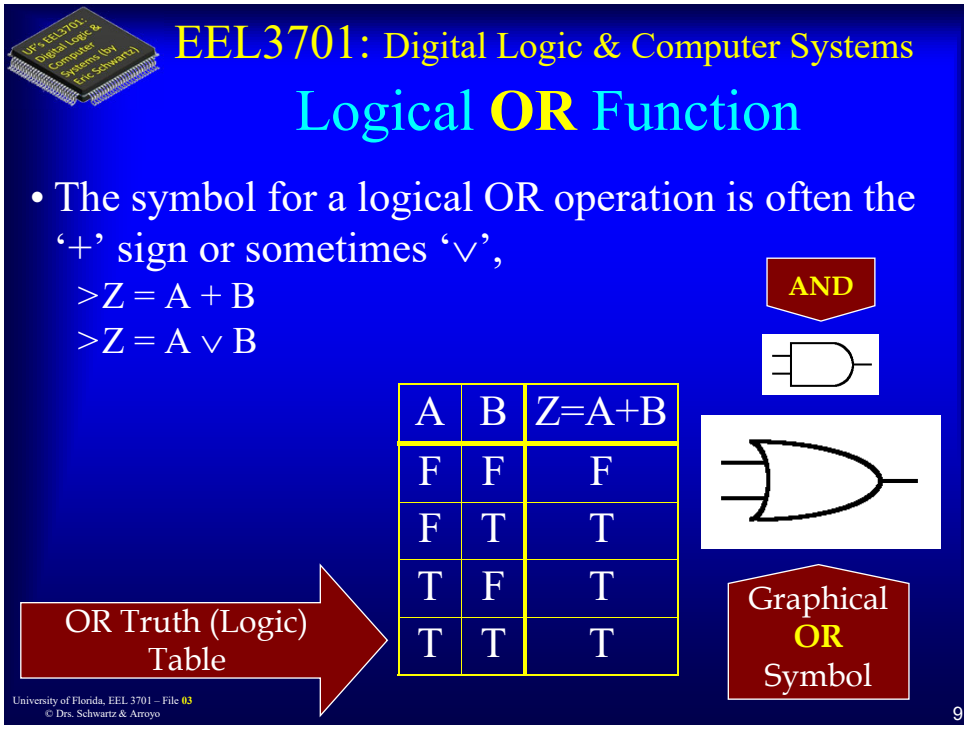
EEL3701: Digital Logic & Computer Systems
Logical OR Function
• The symbol for a logical OR operation is often the '+' sign or sometimes '∨',
>Z = A + B
>Z = A ∨ B
AND
A B Z=A+B
F F F
F T T
T F T
T T T
OR Truth (Logic) Table
Graphical OR Symbol
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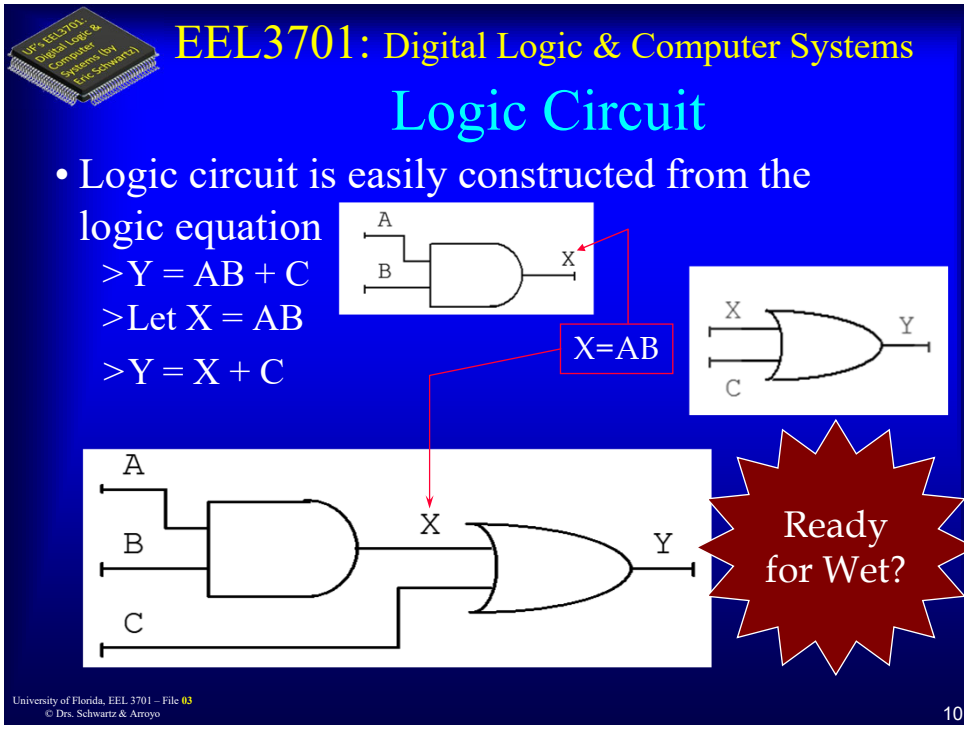
EEL3701: Digital Logic & Computer Systems
Logic Circuit
• Logic circuit is easily constructed from the logic equation
>Y = AB + C
>Let X = AB
>Y = X + C
A
B
X
X=AB
X
Y
C
A
B
C
X
Y
Ready for Wet?
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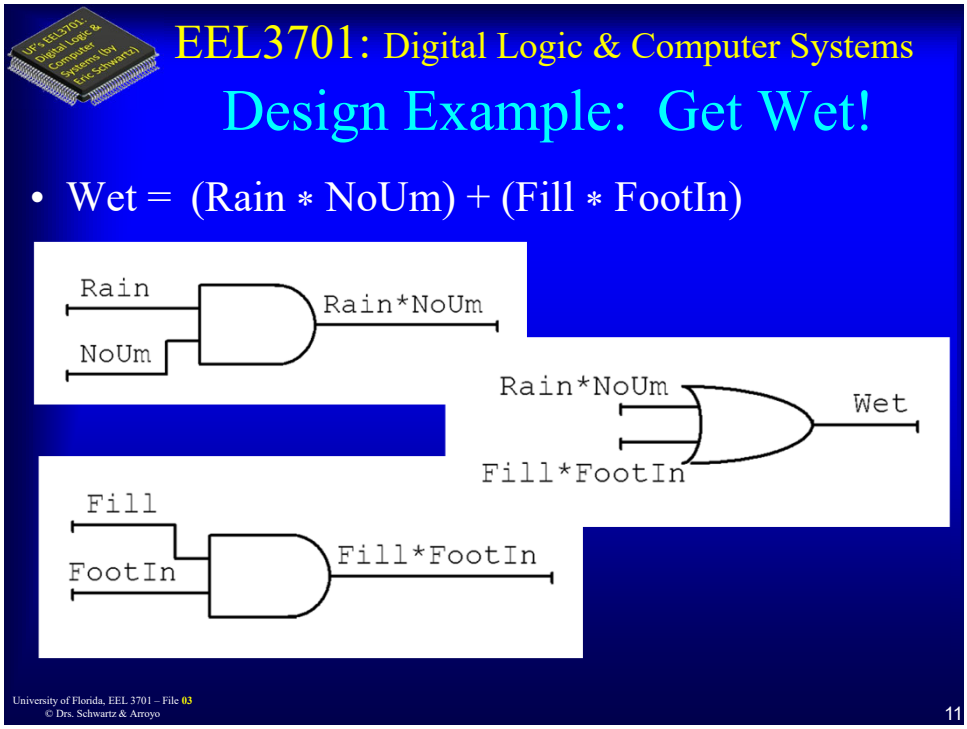
EEL3701: Digital Logic & Computer Systems
Design Example: Get Wet!
• Wet = (Rain ∗ NoUm) + (Fill ∗ FootIn)
Rain
NoUm
Rain*NoUm
Rain*NoUm
Wet
Fill*FootIn
Fill
FootIn
Fill*FootIn
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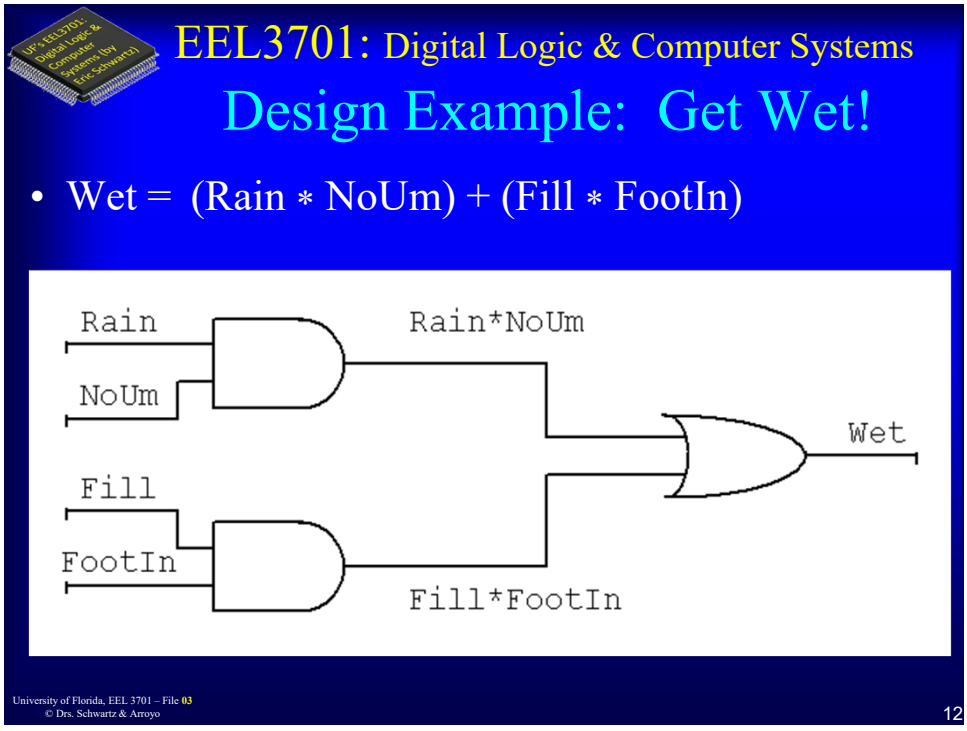
EEL3701: Digital Logic & Computer Systems
Design Example: Get Wet!
• Wet = (Rain ∗ NoUm) + (Fill ∗ FootIn)
Rain
NoUm
Rain*NoUm
Wet
Fill
FootIn
Fill*FootIn
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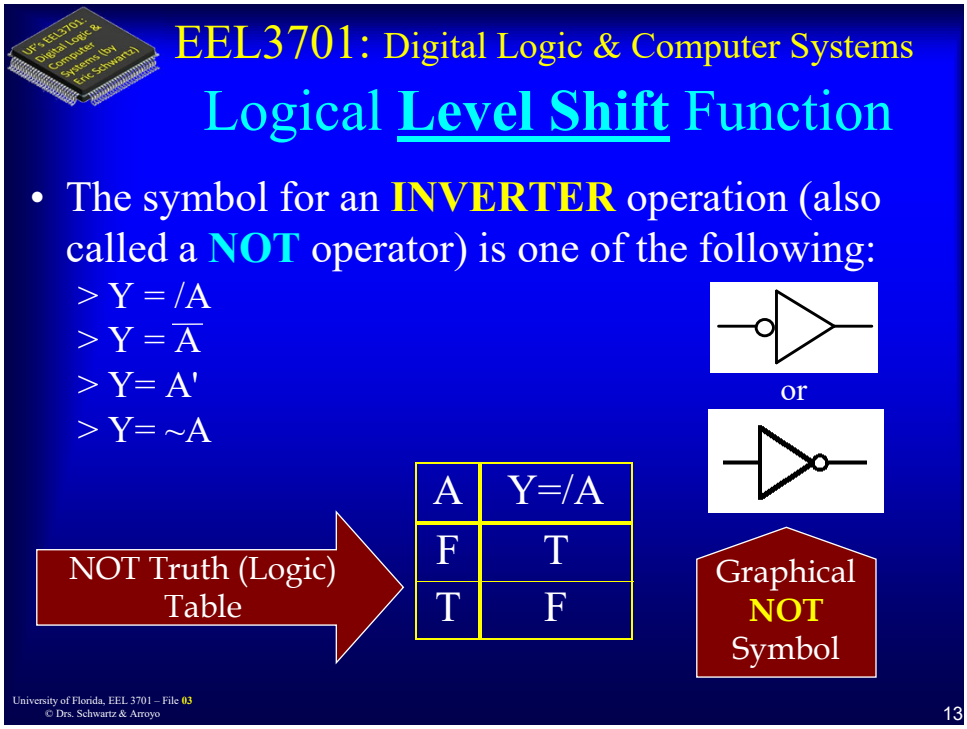
EEL3701: Digital Logic & Computer Systems
Logical Level Shift Function
• The symbol for an INVERTER operation (also called a NOT operator) is one of the following:
> Y = /A
> Y = $\overline{A}$
> Y= A'
> Y= ~A
or
NOT Truth (Logic) Table
A
Y=/A
F
T
T
F
Graphical NOT Symbol
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EEL3701: Digital Logic & Computer Systems
Design Example: Stay Dry!
• We can write a whole new paragraph based on the equation Dry =/Wet
OLD:
When I forget my umbrella and it rains, I get wet. I also get wet when I stick my foot in the bathtub when it has already been filled.
NEW:
I stay dry if I do not get wet!
Wet(H)
Dry(H)
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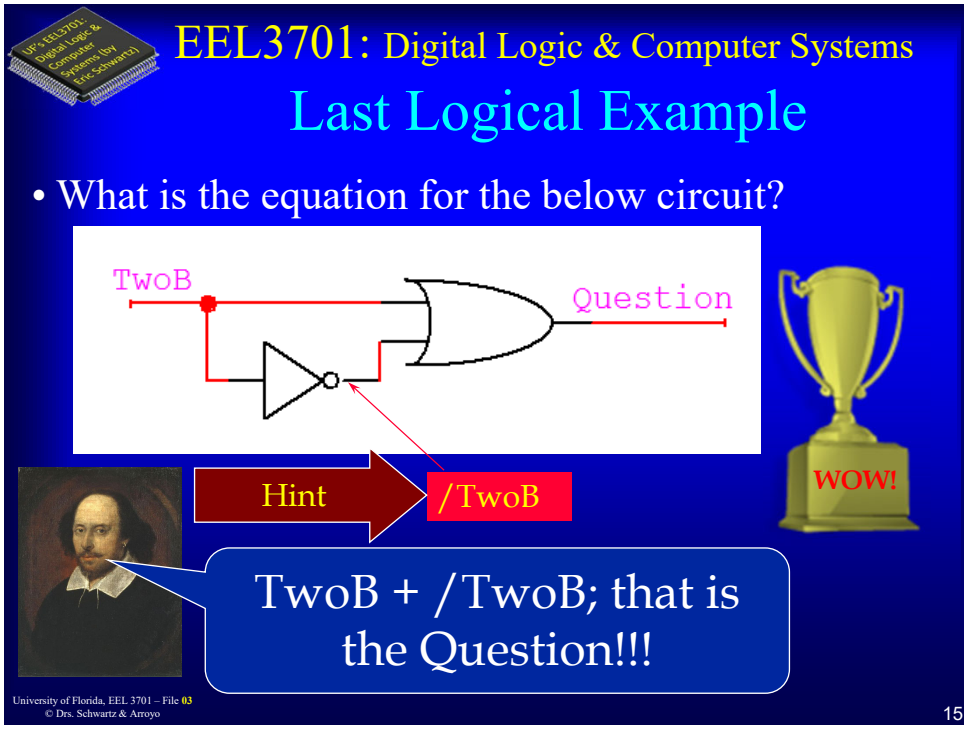
EEL3701: Digital Logic & Computer Systems
Last Logical Example
• What is the equation for the below circuit?
TwoB
Question
WOW!
Hint
/TwoB
TwoB + /TwoB; that is the Question!!!
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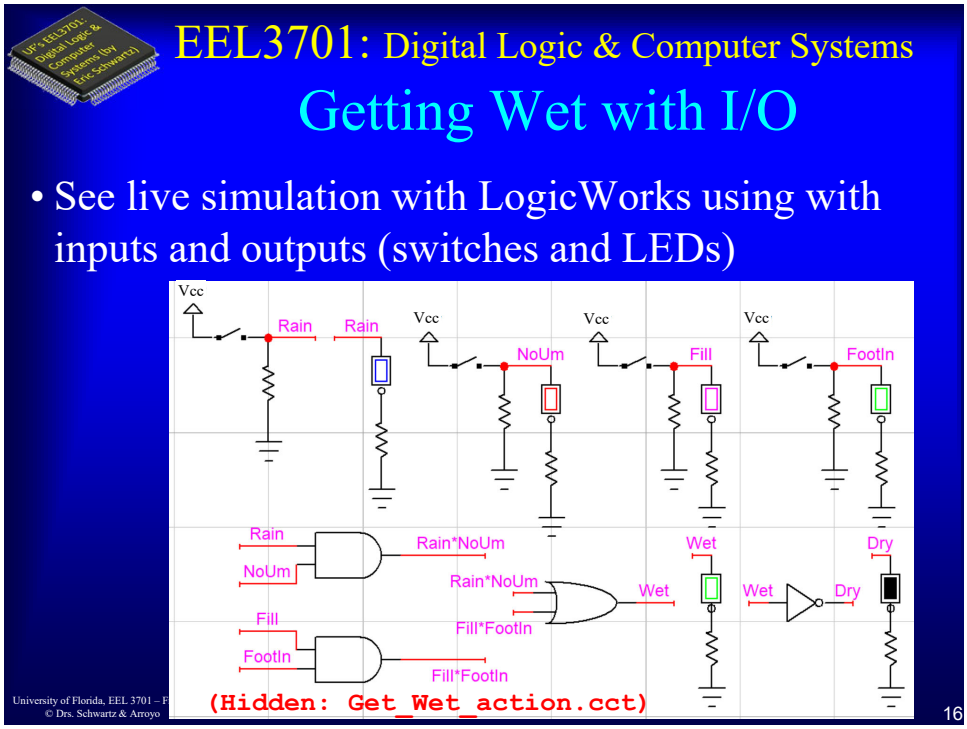
EEL3701: Digital Logic & Computer Systems
Getting Wet with I/O
• See live simulation with LogicWorks using with inputs and outputs (switches and LEDs)
Vcc
Rain
Rain
NoUm
Fill
FootIn
Rain*NoUm
Fill*FootIn
Wet
Dry
(Hidden: Get_Wet_action.cct)
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EEL3701: Digital Logic & Computer Systems
Intro to Logic Design Summary
• Learned to convert a problem statement into a logic equation
• Learned the basic components of a digital circuit: AND, OR, NOT
• Learned to construct a digital circuit from a logic equation
• Learned a little Shakespeare
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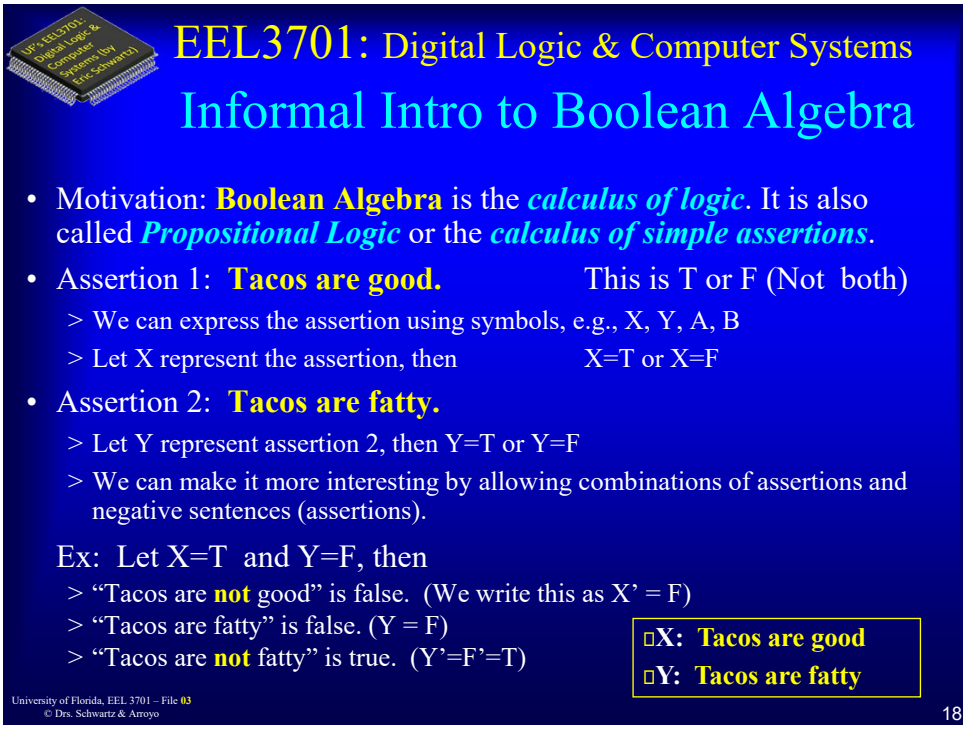
EEL3701: Digital Logic & Computer Systems
Informal Intro to Boolean Algebra
• Motivation: Boolean Algebra is the calculus of logic. It is also called Propositional Logic or the calculus of simple assertions.
• Assertion 1: Tacos are good. This is T or F (Not both)
> We can express the assertion using symbols, e.g., X, Y, A, B
> Let X represent the assertion, then X=T or X=F
• Assertion 2: Tacos are fatty.
> Let Y represent assertion 2, then Y=T or Y=F
> We can make it more interesting by allowing combinations of assertions and negative sentences (assertions).
Ex: Let X=T and Y=F, then
> "Tacos are not good" is false. (We write this as X' = F)
> "Tacos are fatty" is false. (Y = F)
> "Tacos are not fatty" is true. (Y'=F'=T)
X: Tacos are good
Y: Tacos are fatty
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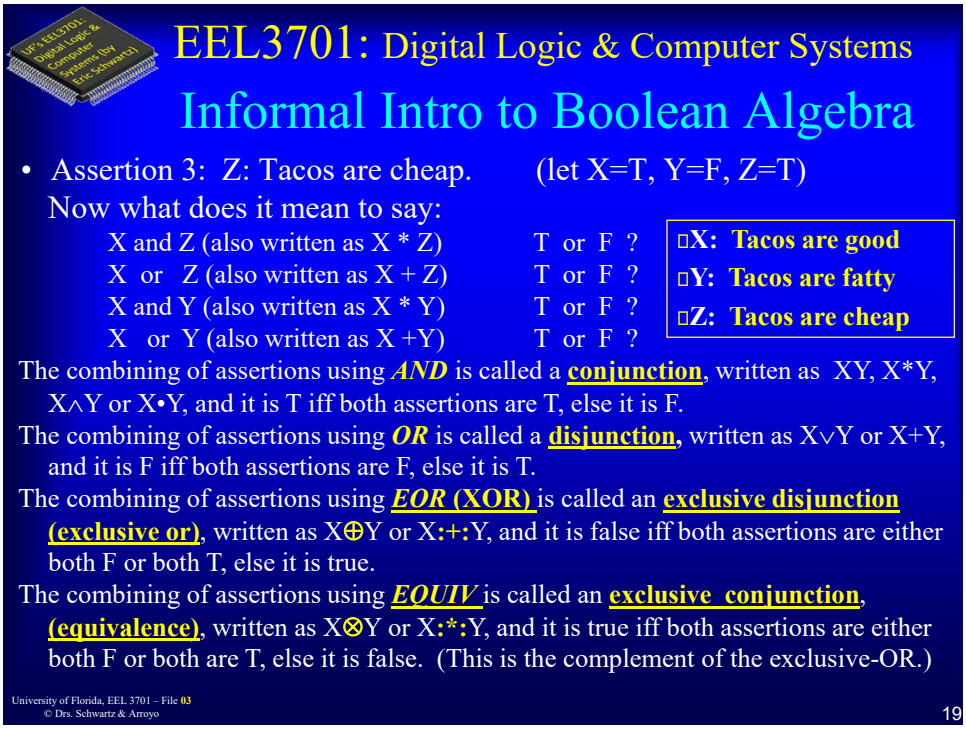
EEL3701: Digital Logic & Computer Systems
Informal Intro to Boolean Algebra
• Assertion 3: Z: Tacos are cheap. (let X=T, Y=F, Z=T)
Now what does it mean to say:
X and Z (also written as X * Z) T or F ?
X or Z (also written as X + Z) T or F ?
X and Y (also written as X * Y) T or F ?
X or Y (also written as X +Y) T or F ?
X: Tacos are good
Y: Tacos are fatty
Z: Tacos are cheap
The combining of assertions using AND is called a conjunction, written as XY, X*Y, X∧Y or X•Y, and it is T iff both assertions are T, else it is F.
The combining of assertions using OR is called a disjunction, written as X∨Y or X+Y, and it is F iff both assertions are F, else it is T.
The combining of assertions using EOR (XOR) is called an exclusive disjunction (exclusive or), written as X⊕Y or X:+:Y, and it is false iff both assertions are either both F or both T, else it is true.
The combining of assertions using EQUIV is called an exclusive conjunction, (equivalence), written as X⊗Y or X:*:Y, and it is true iff both assertions are either both F or both are T, else it is false. (This is the complement of the exclusive-OR.)
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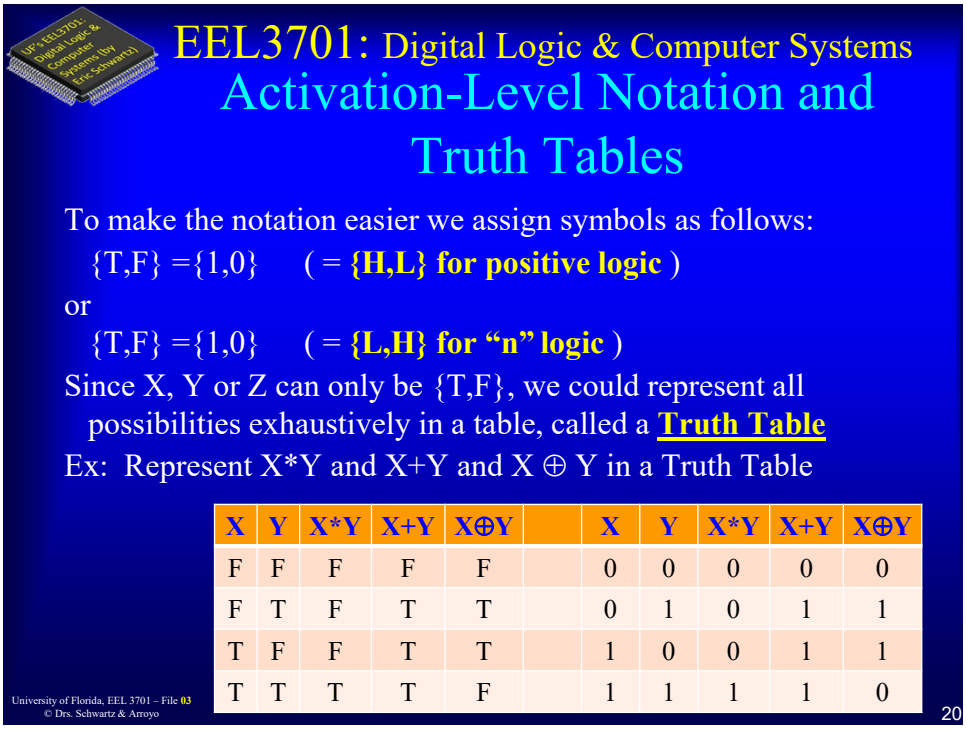
EEL3701: Digital Logic & Computer Systems
Activation-Level Notation and Truth Tables
To make the notation easier we assign symbols as follows:
{T,F} ={1,0} (= {H,L} for positive logic)
or
{T,F} ={1,0} (= {L,H} for "n" logic)
Since X, Y or Z can only be {T,F}, we could represent all possibilities exhaustively in a table, called a Truth Table
Ex: Represent X*Y and X+Y and X ⊕ Y in a Truth Table
X Y X*Y X+Y X⊕Y | X Y X*Y X+Y X⊕Y
F F F F F | 0 0 0 0 0
F T F T T | 0 1 0 1 1
T F F T T | 1 0 0 1 1
T T T T F | 1 1 1 1 0
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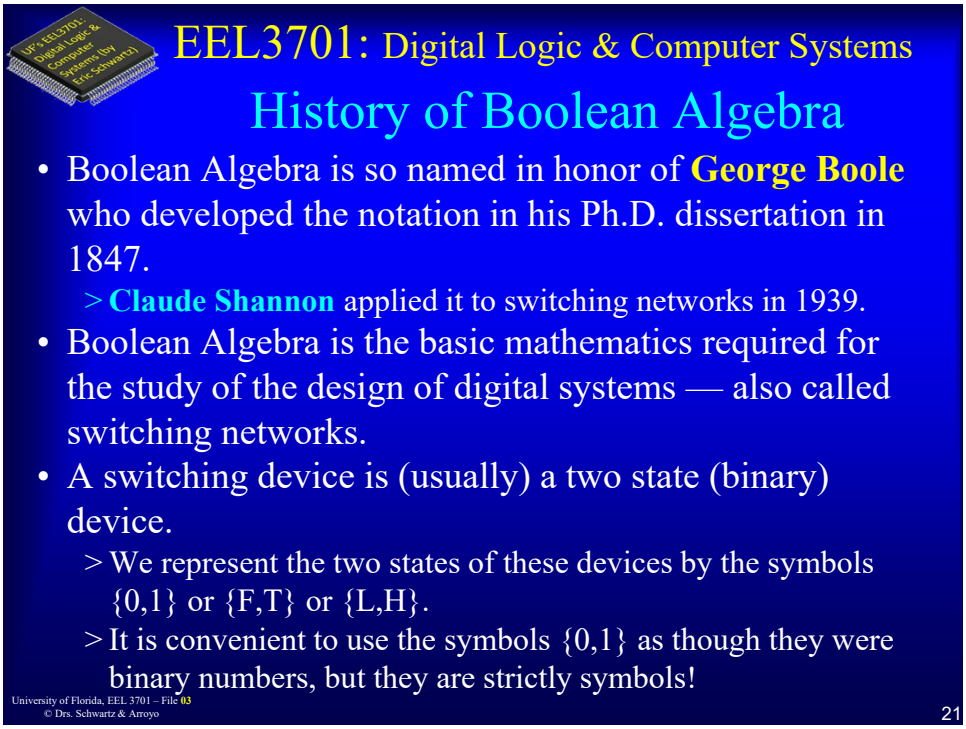
EEL3701: Digital Logic & Computer Systems
History of Boolean Algebra
• Boolean Algebra is so named in honor of George Boole who developed the notation in his Ph.D. dissertation in 1847.
> Claude Shannon applied it to switching networks in 1939.
• Boolean Algebra is the basic mathematics required for the study of the design of digital systems — also called switching networks.
• A switching device is (usually) a two state (binary) device.
> We represent the two states of these devices by the symbols {0,1} or {F,T} or {L,H}.
> It is convenient to use the symbols {0,1} as though they were binary numbers, but they are strictly symbols!
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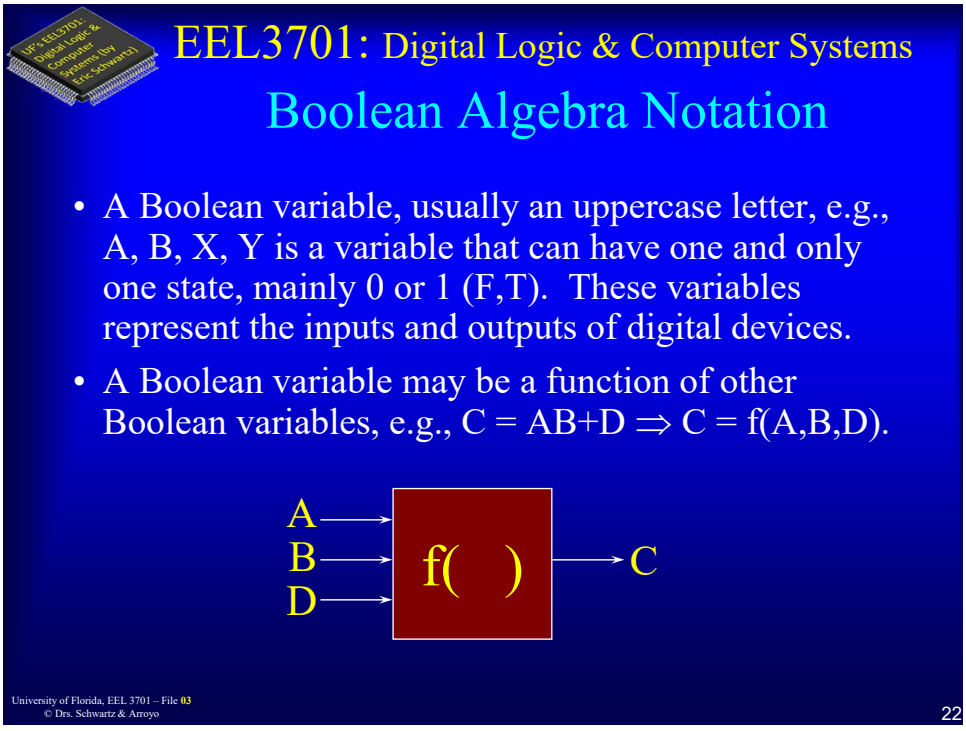
EEL3701: Digital Logic & Computer Systems
Boolean Algebra Notation
• A Boolean variable, usually an uppercase letter, e.g., A, B, X, Y is a variable that can have one and only one state, mainly 0 or 1 (F,T). These variables represent the inputs and outputs of digital devices.
• A Boolean variable may be a function of other Boolean variables, e.g., C = AB+D ⇒ C = f(A,B,D).
A
B
D
f( )
C
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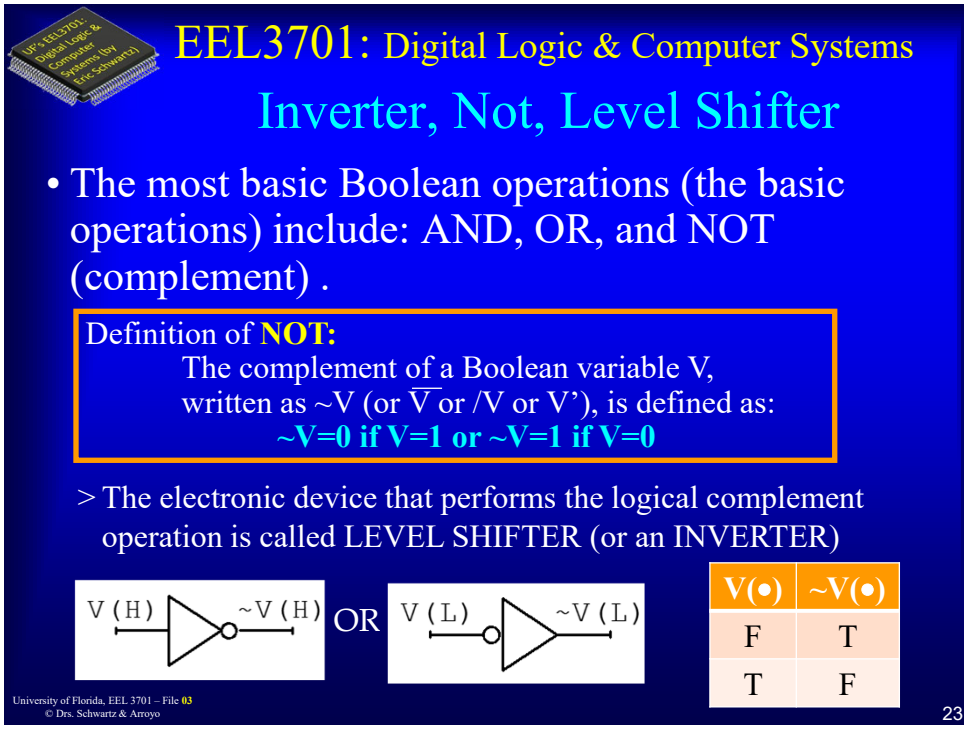
EEL3701: Digital Logic & Computer Systems
Inverter, Not, Level Shifter
• The most basic Boolean operations (the basic operations) include: AND, OR, and NOT (complement) .
Definition of NOT:
The complement of a Boolean variable V, written as ~V (or V̄ or /V or V'), is defined as:
~V=0 if V=1 or ~V=1 if V=0
> The electronic device that performs the logical complement operation is called LEVEL SHIFTER (or an INVERTER)
V(H)
~V(H)
OR
V(L)
~V(L)
V(•) ~V(•)
F T
T F
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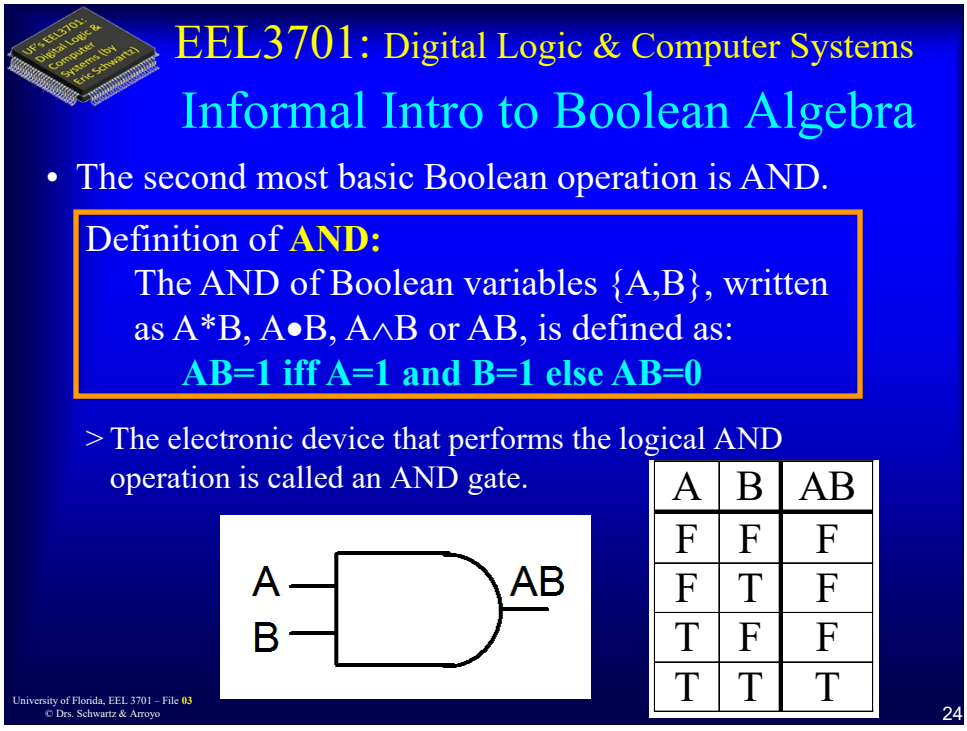
EEL3701: Digital Logic & Computer Systems
Informal Intro to Boolean Algebra
• The second most basic Boolean operation is AND.
Definition of AND:
The AND of Boolean variables {A,B}, written as A*B, A•B, A∧B or AB, is defined as:
AB=1 iff A=1 and B=1 else AB=0
> The electronic device that performs the logical AND operation is called an AND gate.
A
B
AB
A B AB
F F F
F T F
T F F
T T T
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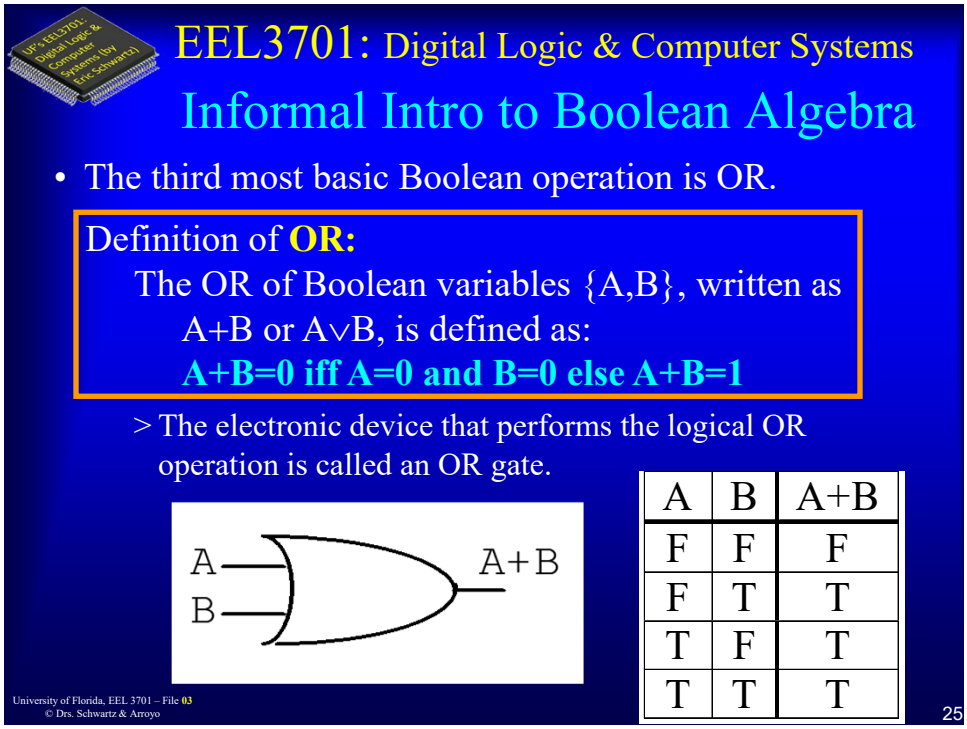
EEL3701: Digital Logic & Computer Systems
Informal Intro to Boolean Algebra
• The third most basic Boolean operation is OR.
Definition of OR:
The OR of Boolean variables {A,B}, written as A+B or A∨B, is defined as:
A+B=0 iff A=0 and B=0 else A+B=1
> The electronic device that performs the logical OR operation is called an OR gate.
A
B
A+B
A B A+B
F F F
F T T
T F T
T T T
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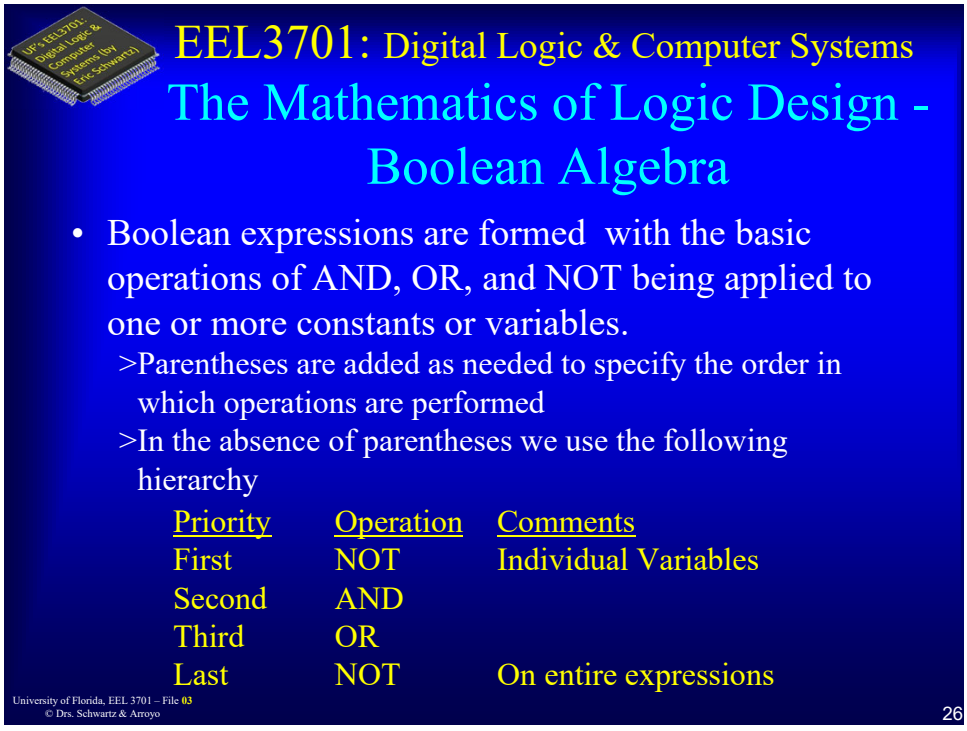
EEL3701: Digital Logic & Computer Systems
The Mathematics of Logic Design - Boolean Algebra
• Boolean expressions are formed with the basic operations of AND, OR, and NOT being applied to one or more constants or variables.
>Parentheses are added as needed to specify the order in which operations are performed
>In the absence of parentheses we use the following hierarchy
Priority Operation Comments
First NOT Individual Variables
Second AND
Third OR
Last NOT On entire expressions
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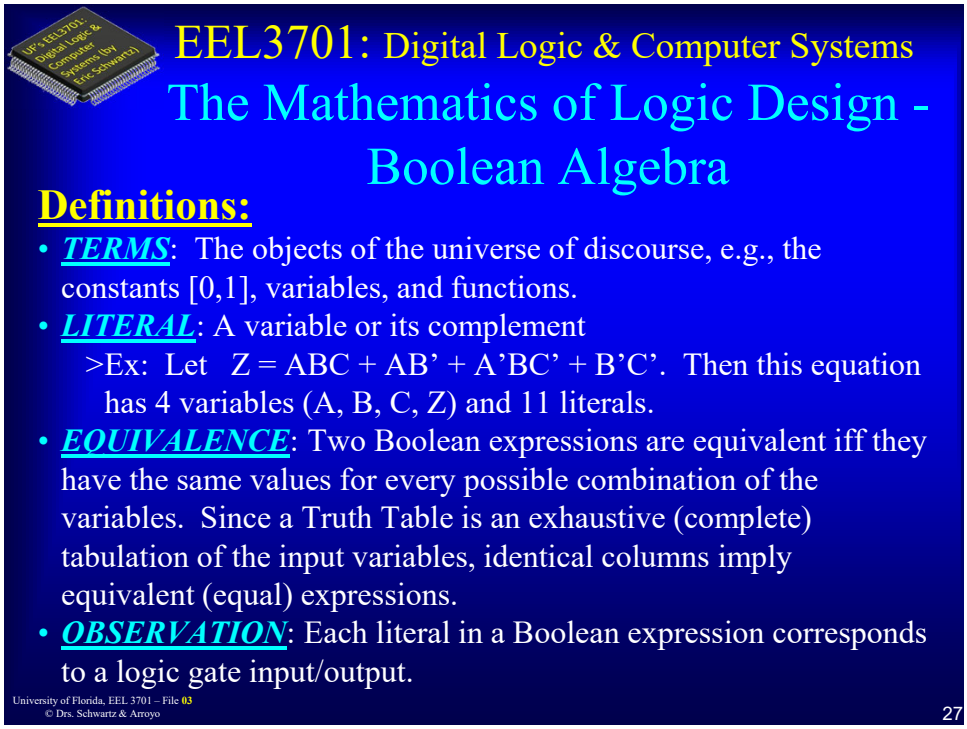
EEL3701: Digital Logic & Computer Systems
The Mathematics of Logic Design - Boolean Algebra
Definitions:
• TERMS: The objects of the universe of discourse, e.g., the constants [0,1], variables, and functions.
• LITERAL: A variable or its complement
>Ex: Let Z = ABC + AB' + A'BC' + B'C'. Then this equation has 4 variables (A, B, C, Z) and 11 literals.
• EQUIVALENCE: Two Boolean expressions are equivalent iff they have the same values for every possible combination of the variables. Since a Truth Table is an exhaustive (complete) tabulation of the input variables, identical columns imply equivalent (equal) expressions.
• OBSERVATION: Each literal in a Boolean expression corresponds to a logic gate input/output.
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The End!
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